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Geometric Phases for Wave Packets of the Landau Problem

Mohamed Nazih Rhimi · Riadh El-Bahi

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Abstract The Landau problem of a charged particle in a plane with a uniform perpendicular magnetic field is analysed in two oscillator modes. The coherent states for the problem have been found out using a general definition of displaced states. The time evolution and the associated nonadiabatic geometric phase for both initially displaced and non-displaced wave packets have been studied. The path integral is derived in a simple way through the calculation of Gaussian integrals via the concept of coherent state wavefunctions.

Keywords Landau problem · Harmonic oscillator · Coherent state · Time evolution · Geometric phase · Wave packet · Gaussian

1 Introduction

The problem of motion of charged particles in two dimensions under the influence of constant perpendicularly applied magnetic field has been early paid attention and has been studied extensively in both classical and quantum views [1–9]. This is essentially a two-dimensional problem since there is no component of the force parallel to the magnetic field and the momentum component along that direction must remain constant [10]. Landau [10] solved the Schrodinger equation in a particular gauge, the now well-known Landau gauge. His solution is a plane wave in one direction and a harmonic oscillator wavefunction in the other direction. Fifty odd years later, Laughlin [4] used a symmetrical gauge to solve the Schrodinger equation paving the way to the study of the fractional quantum Hall effects [5]. Fung [7] analyzed the problem using magnetic translation invariance. In fact, these works are different representations of the same problem.

The Hamiltonian formalism developed by Rhimi and El-Bahi [11] was used to study the Landau problem from both classical and quantum mechanics views [12]. As a matter

M.N. Rhimi (✉)
Preparatory Institute for Engineering Studies (IPEIS), Route Menzel Chaker Km 0.5, 3018 Sfax, Tunisia
e-mail: mrhimi@yahoo.fr

R. El-Bahi
Department of Physics, Faculty of Sciences, University of Sfax, 3018 Sfax, Tunisia

of fact, this formalism proved to be efficient in treating the problem of particle dynamics in a free-electron laser consisting of a helical wiggler magnetic field and a uniform guide field [13–15]. Even though the geometric phases for wave packets of charged particles moving in a constant magnetic field were very seldom considered in the literature [16, 17] because this represents a rather simple and realizable physical situation, they can occur for time dependant systems like spin in a uniform magnetic field.

This paper is a refinement of the work of Rhimi [12]. In Sect. 2, we review the energy levels and the eigenstates of the system. In Sect. 3, the Landau wavefunctions are worked out. Section 4 is devoted to the calculation of the coherent states for our problem. In Sect. 5, we consider the time evolution of an arbitrary wavepacket then find out the nonadiabatic geometric phase for either displaced or non-displaced state. We end by the conclusion.

2 Review of the Model

Consider an electron of charge e and mass m moving on the XY-plane in the presence of a constant perpendicular magnetic field $\mathbf{B_0} = B_0\hat{\mathbf{e}}_\mathbf{z}$. The vector potential that yields the magnetic field is obtained via $\mathbf{B_0} = \nabla \times \mathbf{A_0}$, where we take the symmetric gauge $\mathbf{A_0} = \frac{B_0}{2}(-Y\hat{\mathbf{e}}_X + X\hat{\mathbf{e}}_Y)$. The motion of the electron is governed by the Hamiltonian

$$H = \frac{1}{2m}\left(\mathbf{P} - \frac{e}{c}\mathbf{A_0}\right)^2, \tag{1}$$

which reads for the planar motion

$$H = \frac{1}{2m}(P_X^2 + P_Y^2) + \frac{1}{8}m\omega_0^2(X^2 + Y^2) + \frac{1}{2}\omega_0 L_Z, \tag{2}$$

where $\omega_0 = -eB_0/mc > 0$ and $L_Z = XP_Y - YP_X$. We define

$$\mathbf{A} = \sqrt{\frac{m\omega_0}{8\hbar}}\left[\mathbf{X} - i\mathbf{Y} + \frac{2i}{m\omega_0}(\mathbf{P_X} - i\mathbf{P_Y})\right], \tag{3}$$

$$\mathbf{B} = \sqrt{\frac{m\omega_0}{8\hbar}}\left[\mathbf{X} + i\mathbf{Y} + \frac{2i}{m\omega_0}(\mathbf{P_X} + i\mathbf{P_Y})\right]. \tag{4}$$

The nonvanishing commutators among these operators and their hermitian conjugates are

$$[\mathbf{A}, \mathbf{A}^+] = [\mathbf{B}, \mathbf{B}^+] = \mathbf{1}. \tag{5}$$

The transformed Hamiltonian and angular momentum are then obtained in terms of the density operators explicitly as

$$\mathbf{H} = \hbar\omega_0\left(\mathbf{A}^+\mathbf{A} + \frac{1}{2}\mathbf{1}\right) \equiv \hbar\omega_0\left(\mathbf{N}_{\mathbf{A}^+} + \frac{1}{2}\mathbf{1}\right), \tag{6}$$

and

$$\mathbf{L_Z} = \hbar(\mathbf{A}^+\mathbf{A} - \mathbf{B}^+\mathbf{B}) \equiv \hbar(\mathbf{N}_{\mathbf{A}^+} - \mathbf{N}_{\mathbf{B}^+}) = \hbar(\mathbf{A}\mathbf{A}^+ - \mathbf{B}\mathbf{B}^+) \equiv \hbar(\mathbf{N_A} - \mathbf{N_B}). \tag{7}$$

Our problem is then brought back to a simple harmonic oscillator problem.

The energy levels are

$$E_n = \hbar\omega_0\left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots, \tag{8}$$

and the eigenstates of the density operators satisfy

$$\mathbf{N}_{\mathbf{A}^+}|n, m\rangle = n|n, m\rangle, \qquad \mathbf{N}_{\mathbf{B}^+}|n, m\rangle = m|n, m\rangle, \quad n, m = 0, 1, 2, \ldots, \tag{9}$$

such that we have

$$H|n, m\rangle = E_n|n, m\rangle, \tag{10}$$

and

$$\mathbf{L_Z}|n, m\rangle = (n - m)\hbar|n, m\rangle. \tag{11}$$

The different eigenstates are given by

$$|n, m\rangle = \frac{1}{\sqrt{n!m!}}(\mathbf{A}^+)^n(\mathbf{B}^+)^m|0, 0\rangle, \tag{12}$$

where the ground states satisfy

$$\mathbf{A}|0, 0\rangle = \mathbf{B}|0, 0\rangle = 0. \tag{13}$$

Each Landau level n possesses a ground state mode of oscillation $|n, 0\rangle$ corresponding to the smallest guiding center radius. The ground state mode is subjected to the following conditions

$$\mathbf{B}|n, 0\rangle = 0, \tag{14}$$

$$\mathbf{L_Z}|n, 0\rangle = n\hbar|n, 0\rangle. \tag{15}$$

The guiding center radius increases by applying $\mathbf{B}^+ n$ times to the ground state mode $|n, 0\rangle$. The so obtained mode of oscillation $|n, m\rangle$ is

$$|n, m\rangle = \frac{(\mathbf{B}^+)^m}{\sqrt{m!}}|n, 0\rangle. \tag{16}$$

3 Wavefunctions of the Problem

In the present section, the Landau wavefunctions given by (43) will be worked out using our model. Adopting the Laughlin terminology

$$Z = \sqrt{\frac{m\omega_0}{\hbar}}(\mathbf{X} - i\mathbf{Y})/2, \qquad Z^* = \sqrt{\frac{m\omega_0}{\hbar}}(\mathbf{X} + i\mathbf{Y})/2, \tag{17}$$

where the star denotes complex conjugate and setting m, $\hbar$ and the cyclotron frequency ω_0 to unity, we have

$$\begin{aligned} &\mathbf{A} = \frac{1}{\sqrt{2}}(Z + \partial_{Z^*}), \qquad \mathbf{A}^+ = \frac{1}{\sqrt{2}}(Z^* - \partial_Z), \qquad \mathbf{B} = \frac{1}{\sqrt{2}}(Z^* + \partial_Z), \\ &\mathbf{B}^+ = \frac{1}{\sqrt{2}}(Z - \partial_{Z^*}), \qquad \mathbf{L_Z} = \frac{1}{2}(Z\partial_Z - Z^*\partial_{Z^*}). \end{aligned} \tag{18}$$

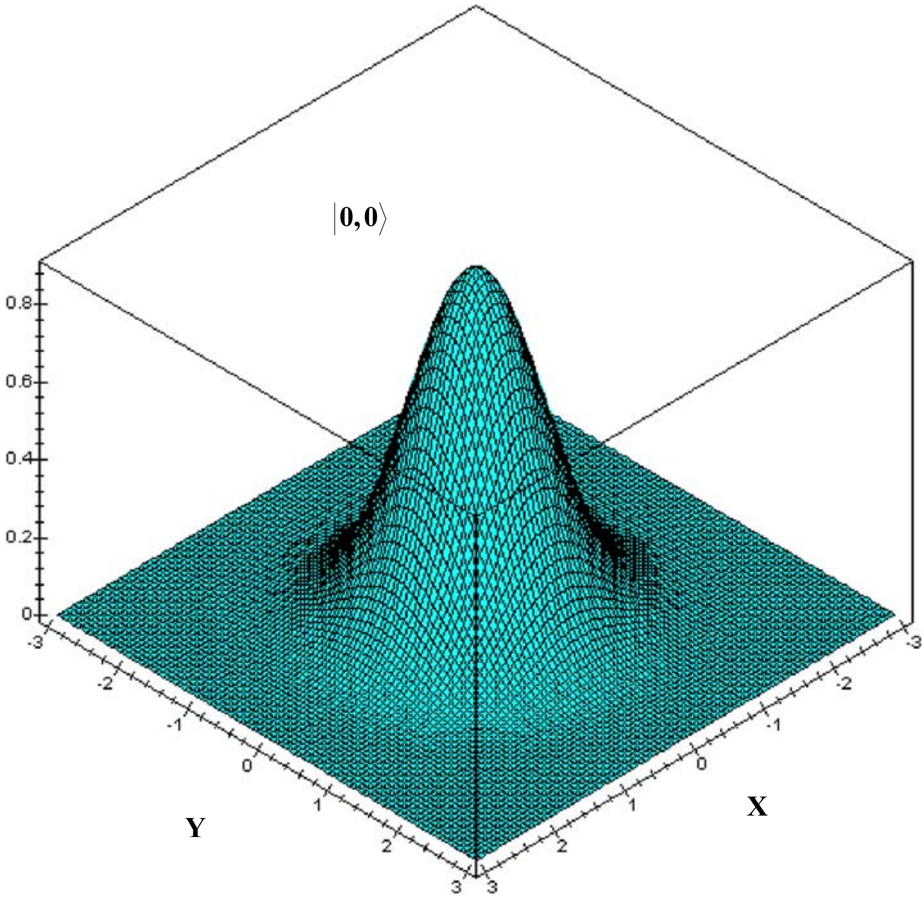

Fig. 1 The wavefunction $|m,n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n=0$ and $m=0$

The ground state of the density operator $\mathbf{N}_{\mathbf{B}^+}$ in each Landau level n is specified by (14) and (15):

$$(Z^* + \partial_Z)|n,0\rangle = 0, \tag{19}$$

$$(Z^*\partial_{Z^*} - Z\partial_Z)|n,0\rangle = 2n|n,0\rangle. \tag{20}$$

The normalized n-th ground state which is the solution of the two coupled equations (19) and (20), is then obtained as

$$|n,0\rangle = \sqrt{\frac{2^{(6n+\frac{1}{2})}(2n)!}{\pi^{1/2}(4n)!}}(Z^*)^{2n}e^{-ZZ^*} = \sqrt{\frac{2^{(2n+\frac{1}{2})}}{\Gamma(2n+\frac{1}{2})}}(Z^*)^{2n}e^{-ZZ^*}, \tag{21}$$

with the zeroth state

$$|0,0\rangle = \left(\frac{2}{\pi}\right)^{\frac{1}{4}}e^{-ZZ^*}. \tag{22}$$

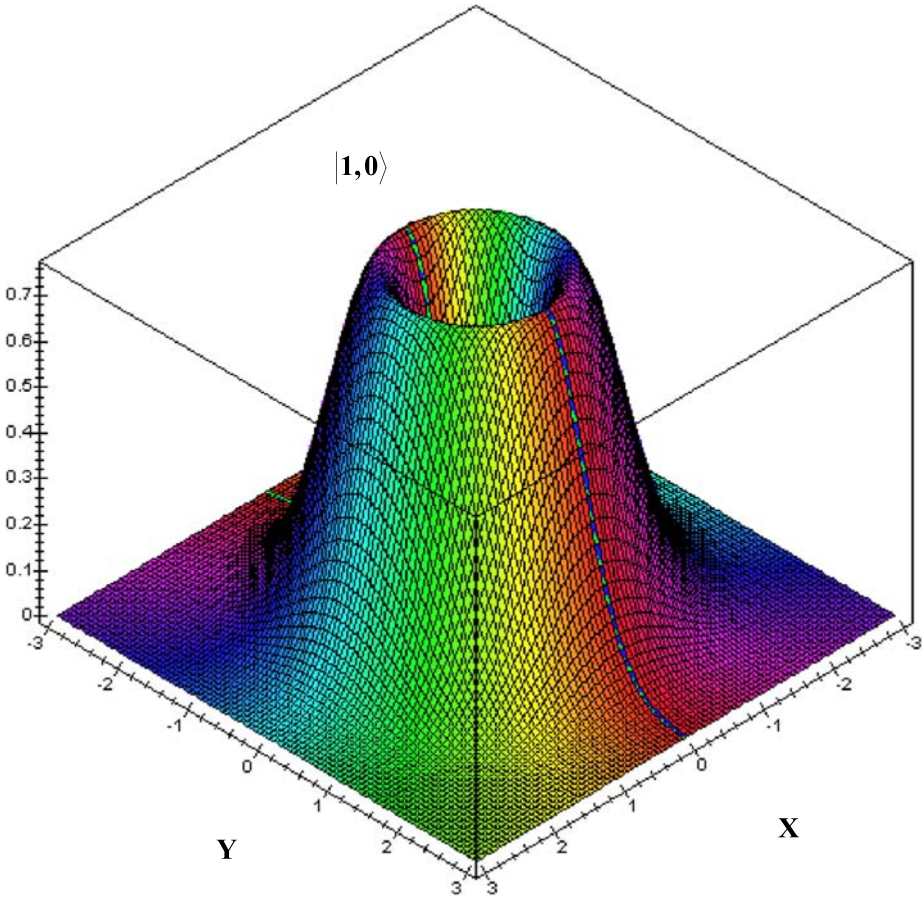

Fig. 2 The wavefunction $|m,n\rangle$ versus $Z\in[-3-3i,3+3i]$, for $n=1$ and $m=0$

The higher states are generated via the application of the rotational $\mathbf{B}^+$

$$|n,m\rangle=\frac{(\mathbf{B}^+)^m}{\sqrt{m!}}|n,0\rangle=\frac{1}{\sqrt{2^m m!}}(Z-\partial_{Z^*})^m|n,0\rangle$$
$$=\frac{1}{\sqrt{2^m m!}}(Z-\partial_{Z^*})^m\left(\sqrt{\frac{2^{(6n+\frac{1}{2})}(2n)!}{\pi^{1/2}(4n)!}}(Z^*)^{2n}e^{-ZZ^*}\right). \tag{23}$$

The states of the lowest Landau level are

$$|0,m\rangle=\frac{2^m Z^m}{\sqrt{m!}}|0,0\rangle=\sqrt{\frac{2^{(2m+\frac{1}{2})}}{\pi^{1/2}m!}}Z^m e^{-ZZ^*}. \tag{24}$$

Three-dimensional plots of the different wave packets are shown in Figs. 1–9, where m and n take the values varying from zero to two.

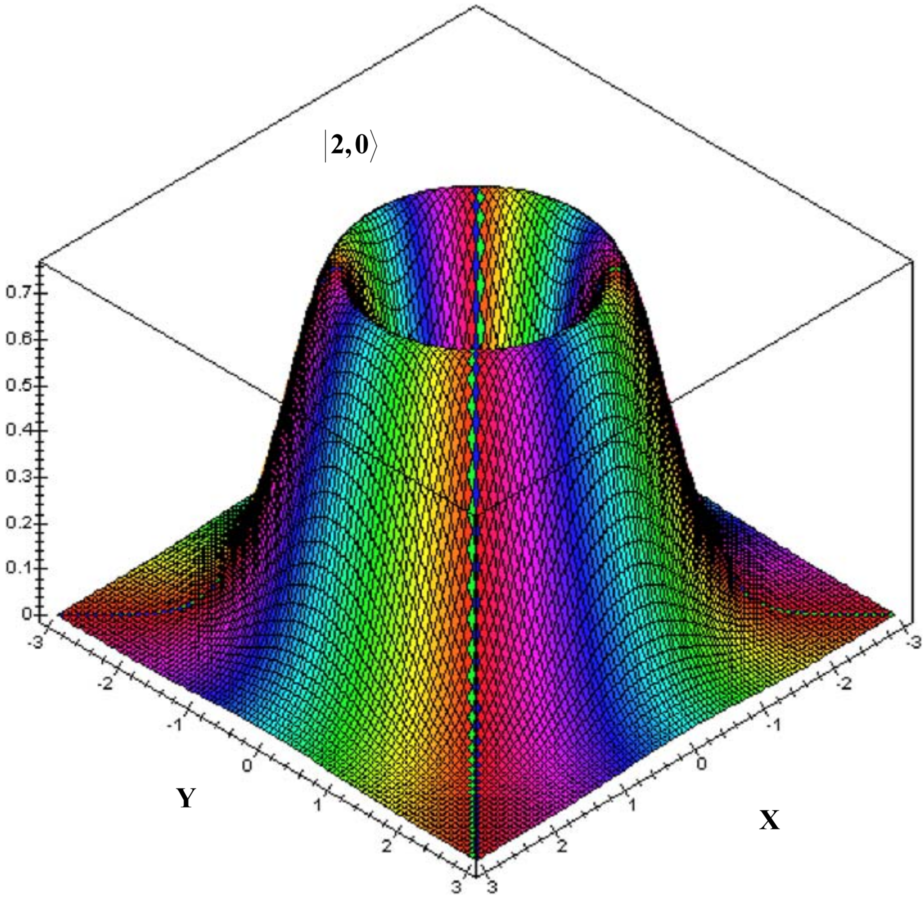

Fig. 3 The wavefunction $|m, n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n=2$ and $m=0$

4 Coherent States

The coherent state $|\alpha, \beta\rangle$ for our problem is the eigenstate for the operators **A** and **B**

$$\mathbf{A}|\alpha, \beta\rangle = \alpha|\alpha, \beta\rangle, \qquad \mathbf{B}|\alpha, \beta\rangle = \beta|\alpha, \beta\rangle, \tag{25}$$

where α and β are complex numbers and the eigenstate $|\alpha, \beta\rangle$ satisfies

$$|\alpha, \beta\rangle = D(\alpha)D(\beta)|0, 0\rangle, \tag{26}$$

with the displacement operators defined as

$$D(\alpha) = \exp(\alpha\mathbf{A}^+ - \alpha^*\mathbf{A}), \qquad D(\beta) = \exp(\beta\mathbf{B}^+ - \beta^*\mathbf{B}). \tag{27}$$

In the z-representation, the system (25) takes the following form

$$\frac{1}{\sqrt{2}}(Z + \partial_{Z^*})|\alpha, \beta\rangle = \alpha|\alpha, \beta\rangle, \qquad \frac{1}{\sqrt{2}}(Z^* + \partial_Z)|\alpha, \beta\rangle = \beta|\alpha, \beta\rangle.$$

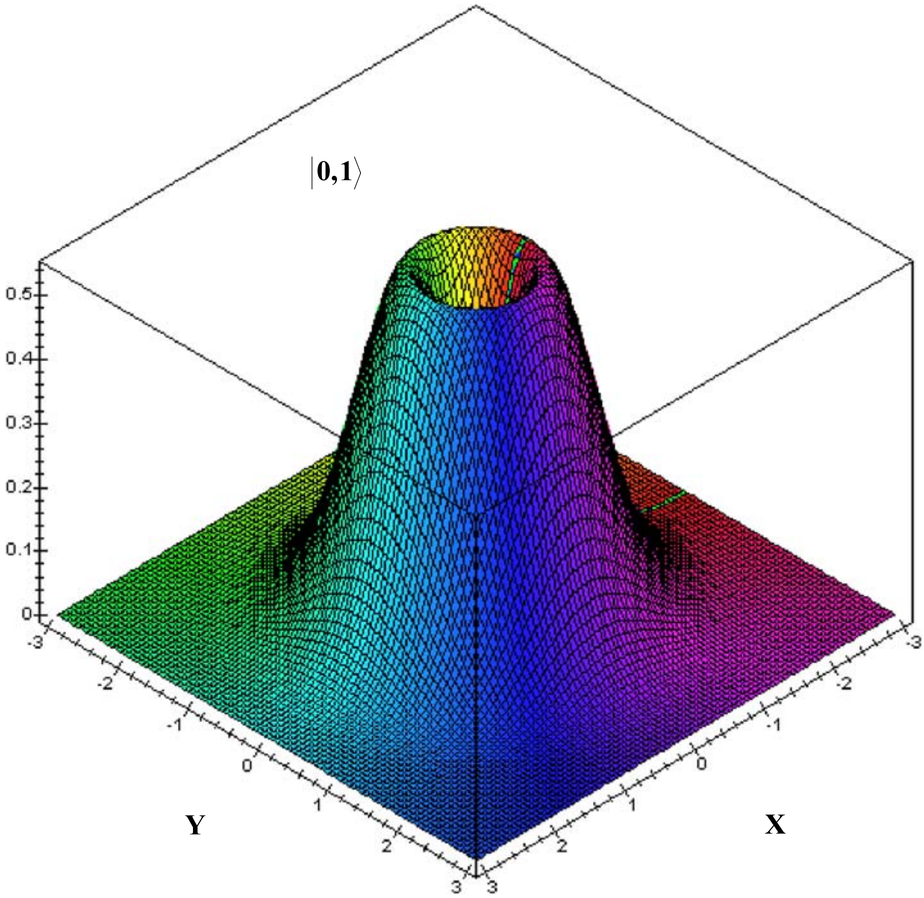

Fig. 4 The wavefunction $|m,n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n=0$ and $m=1$

The normalized solution of the system above is our coherent state.

If we take an arbitrary state $|\psi\rangle$ and act $D(\alpha)$ on it, we obtain the displaced state

$$\langle Z|\psi,\alpha\rangle = \exp\left(\frac{\alpha Z^* - \alpha^* Z}{\sqrt{2}}\right)\left\langle Z - \frac{\alpha}{\sqrt{2}}\middle|\psi\right\rangle, \tag{28}$$

where we have adopted the notation $\langle Z|\psi,\alpha\rangle \equiv \psi_\alpha(Z)$ and $\langle Z-\alpha|\psi\rangle \equiv \psi(Z-\alpha)$.

Equation (28) can be put in another form as

$$\langle X, Y|\psi,\alpha\rangle = \exp\left[\frac{i}{\sqrt{2}}(\alpha_Y X - \alpha_X Y)\right]\langle X - \sqrt{2}\alpha_X, Y - \sqrt{2}\alpha_Y|\psi\rangle, \tag{29}$$

where $\alpha_X = \mathrm{Re}(\alpha)$ and $\alpha_Y = \mathrm{Im}(\alpha)$.

Using the definition of the coherent state for our problem given by (26), we find after some algebra the action of $D(\alpha)D(\beta)$ on the arbitrary state $|\psi\rangle$ as

$$\psi_{\alpha\beta}(Z) \equiv \langle Z|\psi,\alpha,\beta\rangle = \exp\left[\frac{(\alpha-\beta^*)Z^* - (\alpha^*-\beta)Z}{\sqrt{2}}\right]\left\langle Z - \frac{\alpha+\beta^*}{\sqrt{2}}\middle|\psi\right\rangle, \tag{30}$$

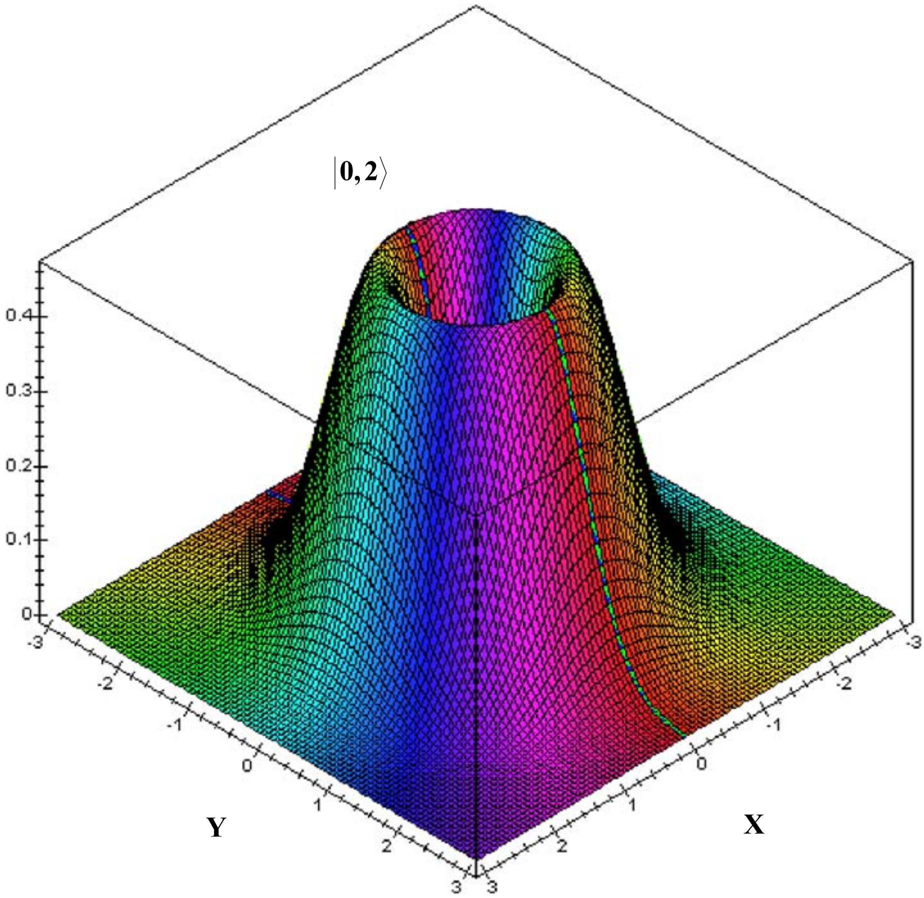

Fig. 5 The wavefunction $|m, n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n = 0$ and $m = 2$

or equivalently

$$\langle X, Y|\psi, \alpha, \beta\rangle = \exp\left[\frac{i}{\sqrt{2}}((\alpha+\beta)_Y X + (\alpha-\beta)_X Y)\right] \times \langle X - \sqrt{2}(\alpha+\beta)_X, Y + \sqrt{2}(\alpha-\beta)_Y|\psi\rangle. \tag{31}$$

Taking the ground state $|\psi\rangle = |0, 0\rangle$ and using (22) and (30), we find

$$\langle Z|0, 0\rangle_{\alpha\beta} = \exp\left[\frac{(\alpha-\beta^*)Z^* - (\alpha^*-\beta)Z}{\sqrt{2}}\right]\left\langle Z - \frac{\alpha+\beta^*}{\sqrt{2}}\middle|0, 0\right\rangle, \tag{32}$$

or

$$\langle Z|0, 0\rangle_{\alpha\beta} = \left(\frac{2}{\pi}\right)^{\frac{1}{4}} e^{[\frac{(\alpha-\beta^*)Z^*-(\alpha^*-\beta)Z}{\sqrt{2}}]} e^{-Z_d Z_d^*}, \tag{33}$$

where $Z_d = Z - \frac{\alpha+\beta^*}{\sqrt{2}}$ and $Z_d^* = Z^* - \frac{\alpha^*+\beta}{\sqrt{2}}$.

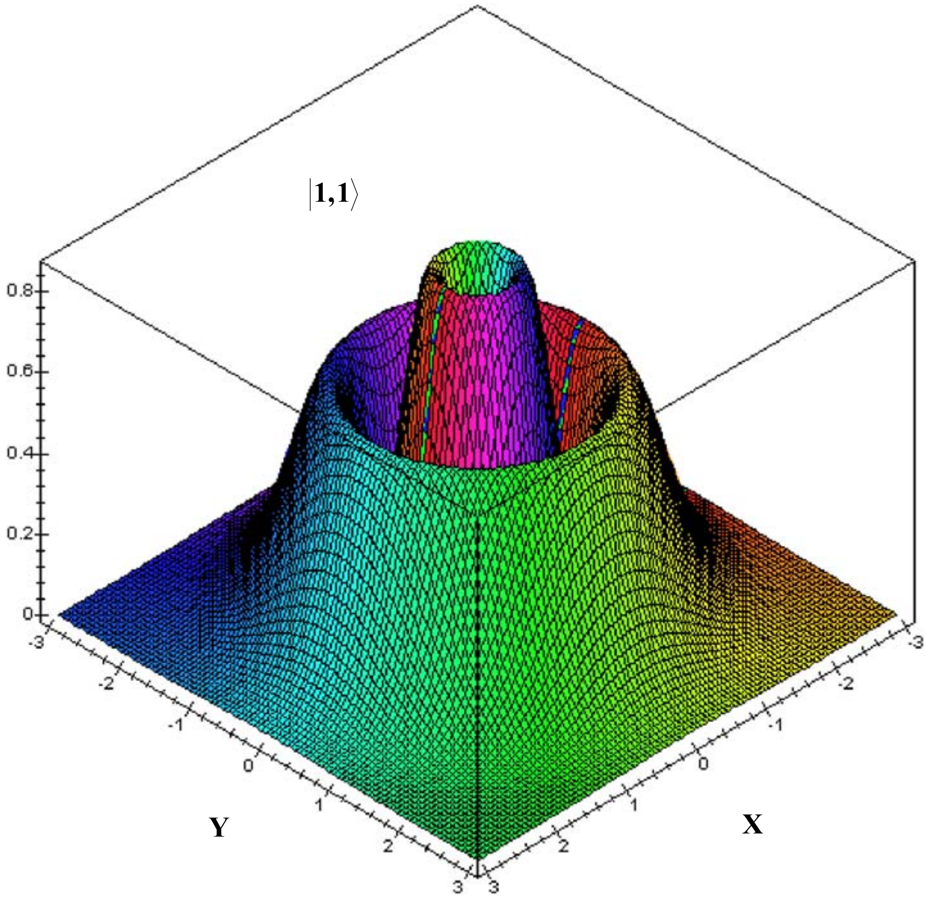

Fig. 6 The wavefunction $|m, n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n = 1$ and $m = 1$

The displaced higher states are obtained via (23) and (30) as

$$\langle Z|n, m\rangle_{\alpha\beta} = e^{[\frac{(\alpha-\beta^*)Z^*-(\alpha^*-\beta)Z}{\sqrt{2}}]} \frac{1}{\sqrt{2^m m!}} (Z_d - \partial_{Z_d^*})^m \left(\sqrt{\frac{2^{(6n+\frac{1}{2})}(2n)!}{\pi^{1/2}(4n)!}} (Z_d^*)^{2n} e^{-Z_d Z_d^*} \right), \quad (34)$$

which can be written shortly as

$$\langle Z|m, n\rangle_{\alpha\beta} = \exp\left[\frac{(\alpha-\beta^*)Z^* - (\alpha^*-\beta)Z}{\sqrt{2}}\right] \left\langle Z - \frac{\alpha+\beta^*}{\sqrt{2}} \middle| m, n \right\rangle. \quad (35)$$

The intermediate states are obtained using (21) and (30) as

$$\langle Z|n, 0\rangle_{\alpha\beta} = e^{[\frac{(\alpha-\beta^*)Z^*-(\alpha^*-\beta)Z}{\sqrt{2}}]} \sqrt{\frac{2^{(6n+\frac{1}{2})}(2n)!}{\pi^{1/2}(4n)!}} (Z_d^*)^{2n} e^{-Z_d Z_d^*}, \quad (36)$$

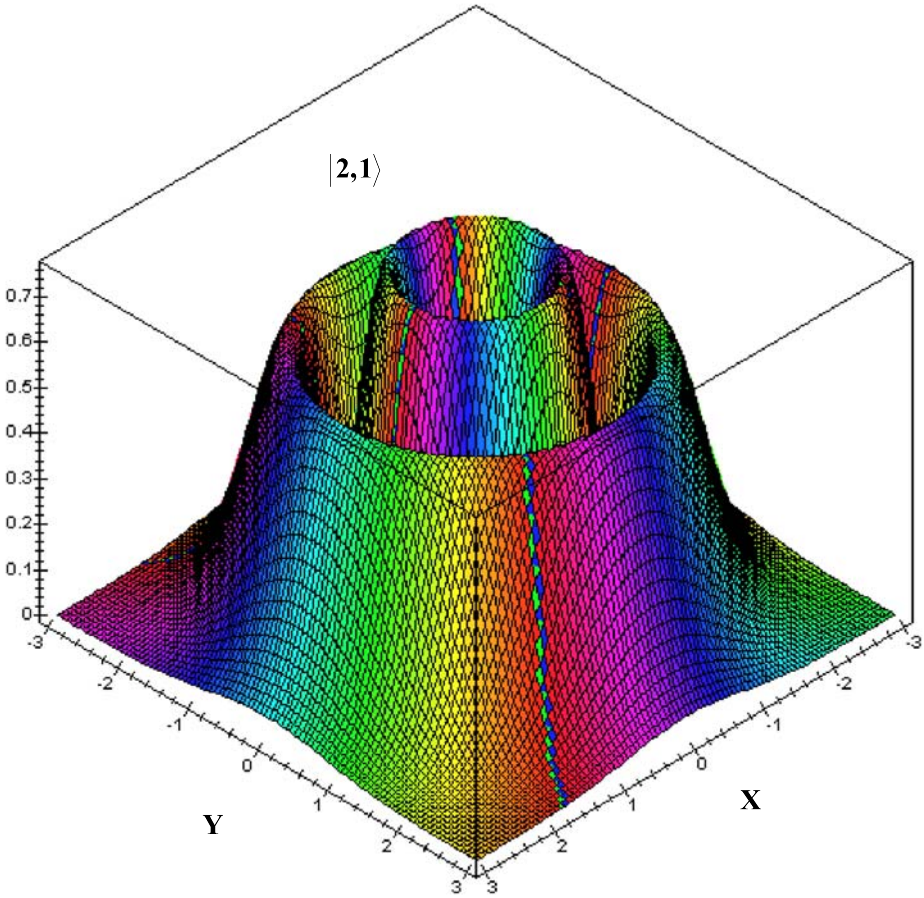

Fig. 7 The wavefunction $|m,n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n=2$ and $m=1$

and (24) and (30) as

$$\langle Z|0,m\rangle_{\alpha\beta} = e^{[\frac{(\alpha-\beta^*)Z^*-(\alpha^*-\beta)Z}{\sqrt{2}}]} \frac{1}{\sqrt{2^m m!}} \left(\sqrt{\frac{2^{(2m+\frac{1}{2})}}{\pi^{1/2} m!}} Z_d^m e^{-Z_d Z_d^*} \right). \tag{37}$$

The coherent state for the lowest level for the quantum number n only is obtained from (28) as

$$\begin{aligned} \langle Z|0\rangle_\alpha &= e^{(\frac{\alpha Z^* - \alpha^* Z}{\sqrt{2}})} \left\langle Z - \frac{\alpha}{\sqrt{2}} \middle| 0 \right\rangle = e^{(\frac{\alpha Z^* - \alpha^* Z}{\sqrt{2}})} \left(\frac{2}{\pi}\right)^{\frac{1}{4}} e^{-(Z-\frac{\alpha}{\sqrt{2}})(Z^*-\frac{\alpha^*}{\sqrt{2}})} \\ &= e^{-\frac{1}{2}\alpha\alpha^* + \sqrt{2}\alpha Z^*} \left(\frac{2}{\pi}\right)^{\frac{1}{4}} e^{-ZZ^*} = e^{-\frac{1}{2}\alpha\alpha^* + \sqrt{2}\alpha Z^*} \langle Z|0\rangle, \end{aligned} \tag{38}$$

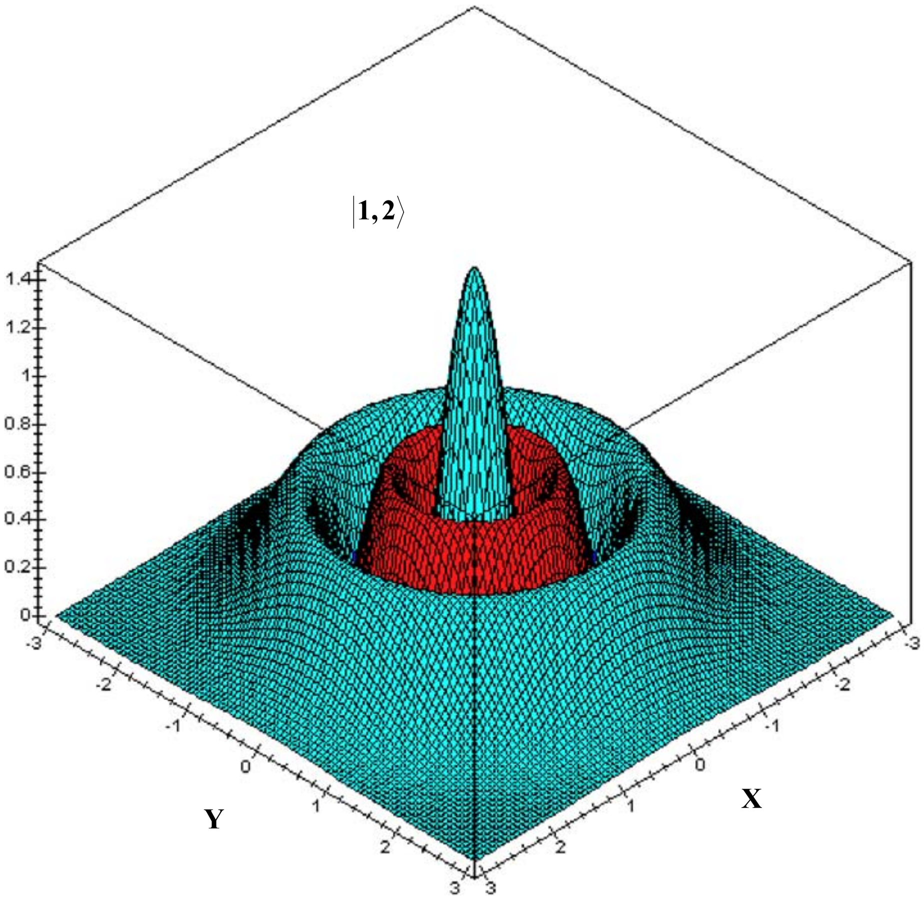

Fig. 8 The wavefunction $|m,n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n=1$ and $m=2$

and from (30) for both numbers n and m as

$$\begin{aligned}
\langle Z|0,0\rangle_{\alpha\beta} &= e^{[\frac{(\alpha-\beta^*)Z^*-(\alpha^*-\beta)Z}{\sqrt{2}}]}\left\langle Z-\frac{\alpha+\beta^*}{\sqrt{2}}\middle|0,0\right\rangle \\
&= e^{[\frac{(\alpha-\beta^*)Z^*-(\alpha^*-\beta)Z}{\sqrt{2}}]}\left(\frac{2}{\pi}\right)^{\frac{1}{4}} e^{-(Z-\frac{\alpha+\beta^*}{\sqrt{2}})(Z^*-\frac{\alpha^*+\beta}{\sqrt{2}})} \\
&= e^{-\frac{1}{2}\alpha^*\beta^*} e^{-\frac{1}{2}\alpha\alpha^*+\sqrt{2}\alpha Z^*} e^{-\frac{1}{2}\beta\beta^*+\sqrt{2}\beta(Z-\frac{\sqrt{2}}{4}\alpha)}\left(\frac{2}{\pi}\right)^{\frac{1}{4}} e^{-ZZ^*} \\
&= e^{-\frac{1}{2}\alpha^*\beta^*} e^{-\frac{1}{2}\alpha\alpha^*+\sqrt{2}\alpha Z^*} e^{-\frac{1}{2}\beta\beta^*+\sqrt{2}\beta(Z-\frac{\sqrt{2}}{4}\alpha)}\langle Z|0,0\rangle. \qquad (39)
\end{aligned}$$

Three-dimensional plots of the ground-state displaced wave packet are shown in Figs. 10–13 for different α's and β's.

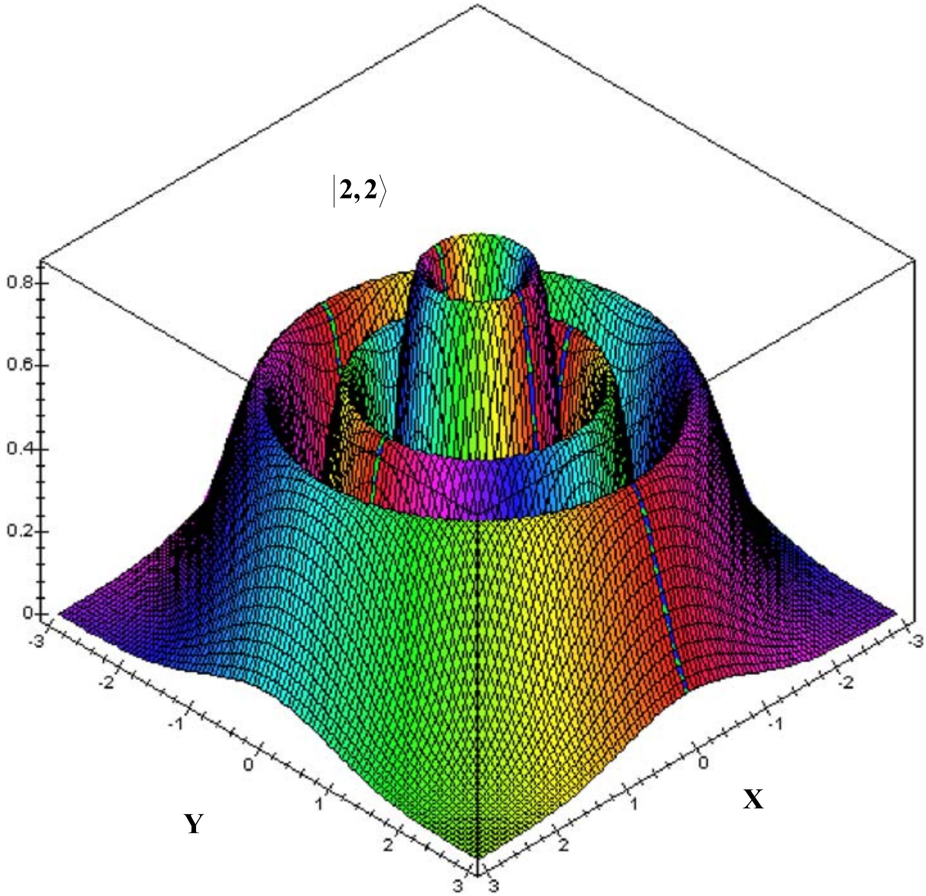

Fig. 9 The wavefunction $|m,n\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $n=2$ and $m=2$

5 Time Evolution and Geometric Phases

Consider the time evolution operator

$$U \equiv U(t) = e^{-iHt},$$

for an arbitrary wave packet $|\psi(t=0)\rangle \equiv |\psi(0)\rangle$, we have

$$|\psi(t)\rangle = U|\psi(0)\rangle,$$

such that

$$\langle Z'|U|Z\rangle = \langle Z'|U|\psi(0)\rangle\langle\psi(0)|Z\rangle = e^{-\frac{it}{2}}\langle Z'|\psi(0)\rangle\langle\psi(0)|Z\rangle = e^{-\frac{it}{2}}\langle Z'|Z\rangle. \tag{40}$$

At the time $T = 2\pi$, we have

$$|\psi(T)\rangle = -|\psi(0)\rangle, \tag{41}$$

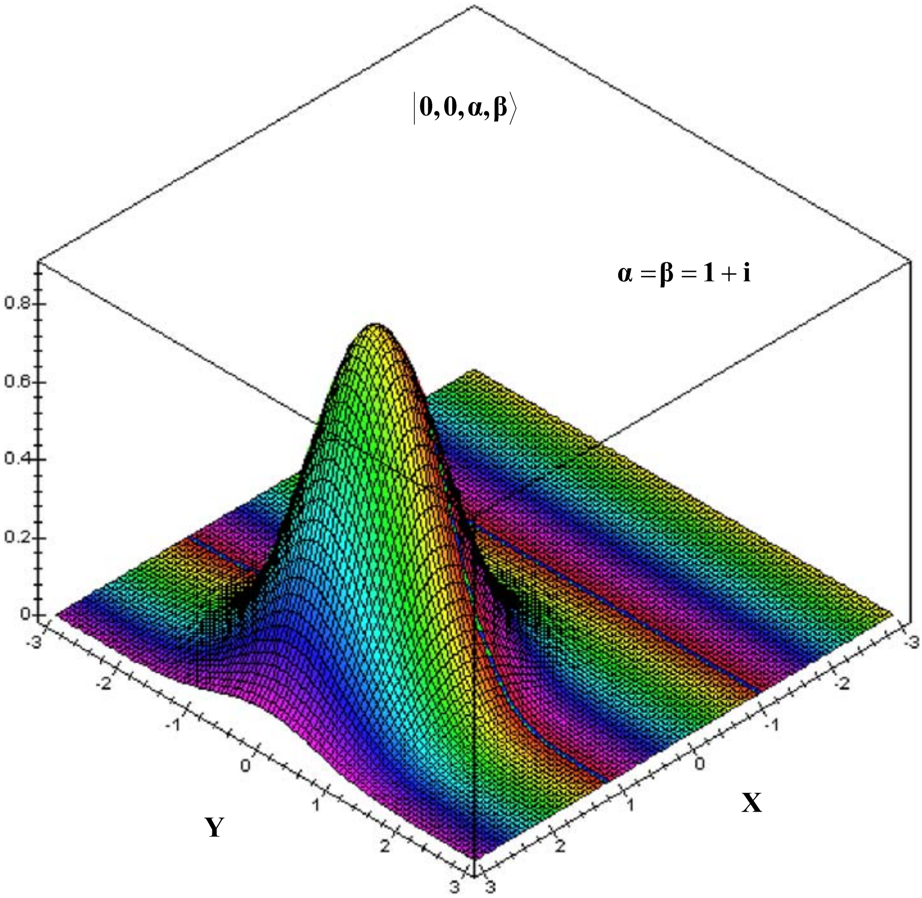

Fig. 10 The displaced ground state wavefunction $|0,0,\alpha,\beta\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $\alpha=\beta=1+i$

and the phase change in a cycle is

$$\varsigma = -\pi \mod 2\pi. \tag{42}$$

To find out the dynamic phase, we evaluate

$$\langle\psi(t)|H|\psi(t)\rangle = \left(\langle \mathbf{A}^{+}\mathbf{A}\rangle + \frac{1}{2}\right) = \langle\psi(0)|H|\psi(0)\rangle, \tag{43}$$

to get

$$\xi = -\int_0^T \langle\psi(t)|H|\psi(t)\rangle dt = -\int_0^T (\langle \mathbf{A}^{+}\mathbf{A}\rangle + 1/2)dt = -\pi(1+2\langle \mathbf{A}^{+}\mathbf{A}\rangle), \tag{44}$$

so that the non-adiabatic geometric phase is

$$\nu = \varsigma - \xi = 2\pi\langle \mathbf{A}^{+}\mathbf{A}\rangle. \tag{45}$$

It is proportional to the mean value of the number operator $\mathbf{N}_{\mathbf{A}^{+}}$.

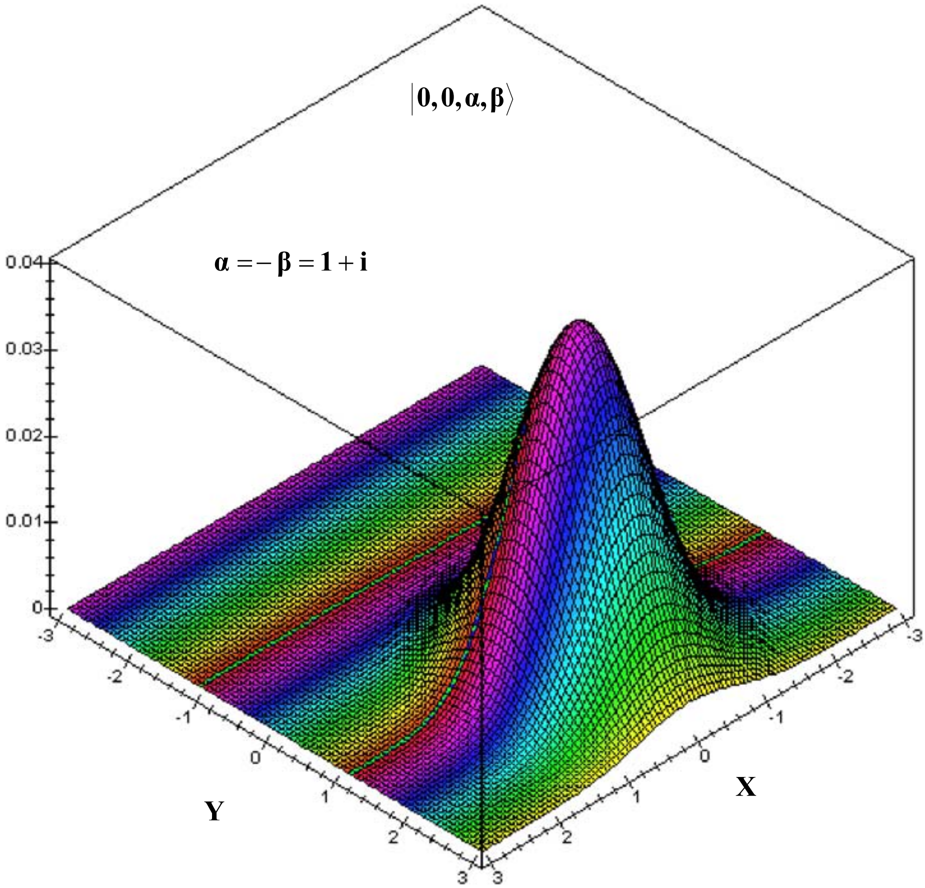

Fig. 11 The displaced ground state wavefunction $|0, 0, \alpha, \beta\rangle$ versus $Z \in [-3 - 3i, 3 + 3i]$, for $\alpha = -\beta = 1 + i$

For a displaced wave packet $|\psi(t = 0), \alpha, \beta\rangle \equiv |\psi(0), \alpha, \beta\rangle$, we have

$$|\psi(t), \alpha, \beta\rangle = U|\psi(0), \alpha, \beta\rangle = e^{-\frac{it}{2}}|\psi(0), \alpha e^{-it}, \beta\rangle, \tag{46}$$

such that we have the finite time propagator

$$\langle Z'|U|Z\rangle = \langle Z'|U|\psi(0), \alpha, \beta\rangle\langle\psi(0), \alpha, \beta|Z\rangle = e^{-\frac{it}{2}}\langle Z'|\psi(0), \alpha e^{-it}, \beta\rangle\langle\psi(0), \alpha, \beta|Z\rangle, \tag{47}$$

where we sum over all the intermediate states α and β. The result of the Gaussian integration is found to be [18]

$$\frac{1}{4\pi i \sin(t/2)} \exp[i(Z' - Z)(Z'^* - Z^*)\cot(t/2) + (Z'Z^* - ZZ'^*)]. \tag{48}$$

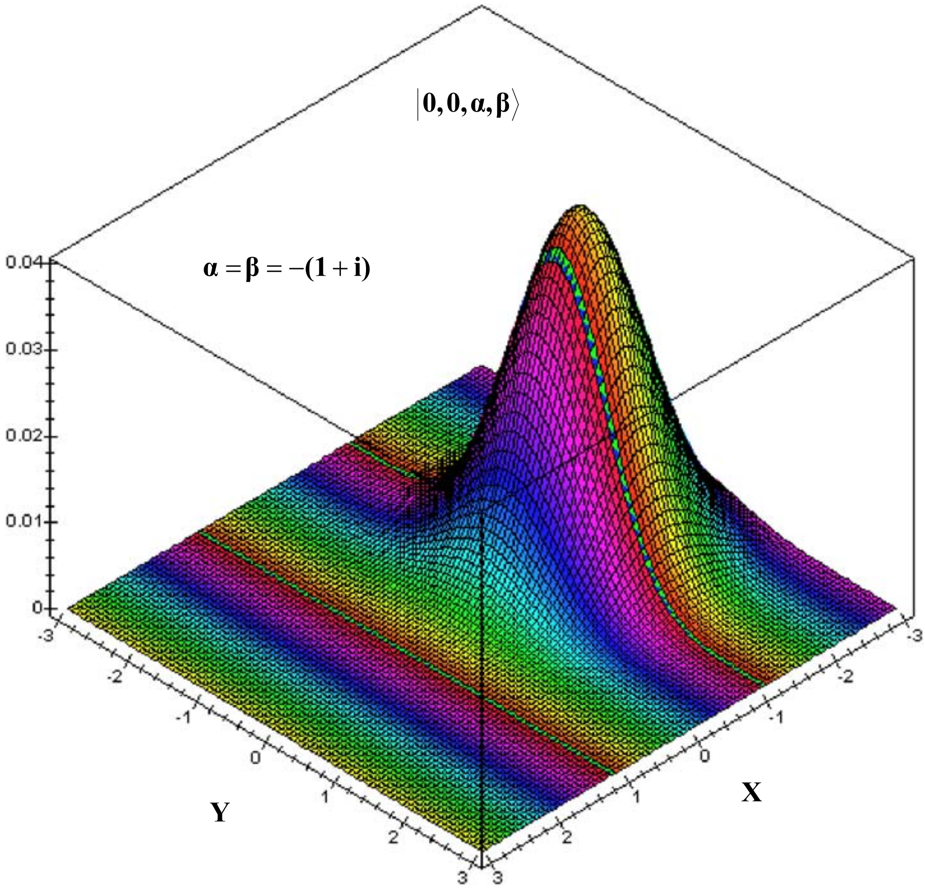

Fig. 12 The displaced ground state wavefunction $|0, 0, \alpha, \beta\rangle$ versus $Z \in [-3 - 3i, 3 + 3i]$, for $\alpha = \beta = -(1 + i)$

On the other hand, we get from (30)

$$\langle Z'|\psi(0), \alpha e^{-it}, \beta\rangle = \exp\left[\frac{(\alpha e^{-it} - \beta^*)Z'^* - (\alpha^* e^{it} - \beta)Z'}{\sqrt{2}}\right]\left\langle Z' - \frac{\alpha e^{-it} + \beta^*}{\sqrt{2}}\middle|\psi(0)\right\rangle, \tag{49}$$

and

$$\langle\psi(0), \alpha, \beta|Z\rangle = \exp\left[\frac{(\alpha^* - \beta)Z - (\alpha - \beta^*)Z^*}{\sqrt{2}}\right]\left\langle\psi(0)\middle|Z^* - \frac{\alpha^* + \beta}{\sqrt{2}}\right\rangle, \tag{50}$$

which substituted in (47), gives

$$\langle Z'|U|Z\rangle = e^{-\frac{it}{2}} e^{[\frac{(\alpha e^{-it} - \beta^*)Z'^* - (\alpha^* e^{it} - \beta)Z'}{\sqrt{2}}]} e^{[\frac{(\alpha^* - \beta)Z - (\alpha - \beta^*)Z^*}{\sqrt{2}}]}\left\langle Z' - \frac{\alpha e^{-it} + \beta^*}{\sqrt{2}}\,\middle|\, Z^* - \frac{\alpha^* + \beta}{\sqrt{2}}\right\rangle. \tag{51}$$

At the time $T = 2\pi$, we have

$$|\psi(T), \alpha, \beta\rangle = -|\psi(0), \alpha, \beta\rangle, \tag{52}$$

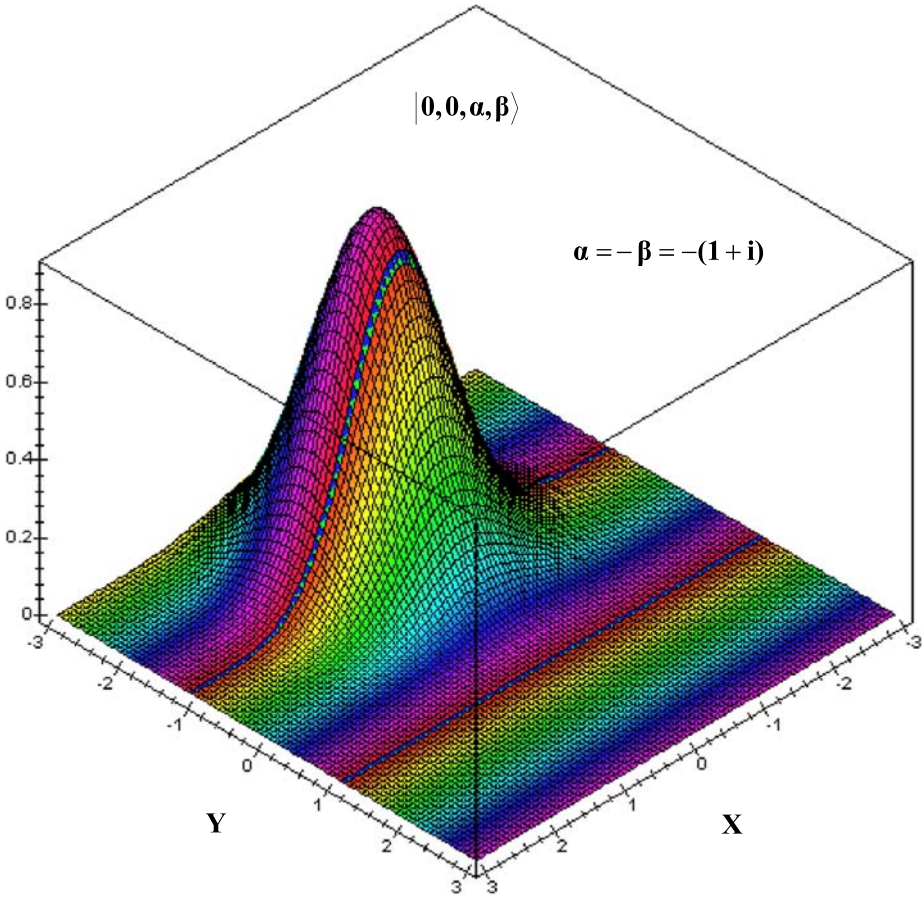

Fig. 13 The displaced ground state wavefunction $|0,0,\alpha,\beta\rangle$ versus $Z \in [-3-3i, 3+3i]$, for $\alpha = -\beta = -(1+i)$

and the phase change of the displaced state's cycle is

$$\varsigma_d = -\pi \quad \mod 2\pi. \tag{53}$$

To find out the dynamic phase of the displaced state, we evaluate

$$\langle\psi(t),\alpha,\beta|H|\psi(t),\alpha,\beta\rangle = \left(\alpha^*\alpha + \frac{1}{2}\right) = \langle\psi(0),\alpha,\beta|H|\psi(0),\alpha,\beta\rangle, \tag{54}$$

to get

$$\xi_d = -\int_0^T \langle\psi(t),\alpha,\beta|H|\psi(t),\alpha,\beta\rangle dt = -\int_0^T (\alpha^*\alpha + 1/2)dt = -\pi(1+2\alpha^*\alpha), \tag{55}$$

so that the non adiabatic geometric phase is

$$\nu_d = \varsigma_d - \xi_d = 2\pi\alpha^*\alpha. \tag{56}$$

It is proportional to the square of the modulus of the eigenvalue of the operator **A** (see (25)).

6 Conclusion

In this paper, we have studied the motion of a charged particle in a constant magnetic field (Landau Problem). The problem is brought to a two-oscillator system that led to the determination of the wavefunctions. The coherent states for the problem have been found out using a general definition of displaced states. The time evolution and the associated nonadiabatic geometric phase for both initially displaced and non-displaced wave packets have been studied. The path integral is derived in a simple way through the calculation of Gaussian integrals via the concept of coherent state wavefunctions. It is important to extend this operator approach to the many body problem which is relevant for the quantum hall effects and to the inclusion of the squeezed states.

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